Approximating Relative Match-Bounds

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Motivation

The 595 problems from TPDB/SRS_STANDARD/ICFP_2010 are

- large: avg. 70 rules of size 2340 (non-ICFP: 3.3 of size 21.5)
- time consuming: VBS CPU time at termCOMP'21
  avg. 90", median 28" (non-ICFP: avg. 51", median 6")
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termCOMP'21 versus '22

	Matchbox	MnM	VBS
termCOMP'21	510	417	514
termCOMP'22	595	594	595

Overview

Methods from this talk (timeout 10")

	rb	**rel. rb**	**mb**	**rel. mb**
solved	370	568	588	590
%	62.2	95.5	98.8	99.2
avg. CPU time	0.29"	0.88"	1.37"	0.93"

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combined with weights + reversal; iterated for rel.

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- Idea: remove relatively (on RFC) match-bounded rules (H/W'10)
- New: approximate this property fast
- Ingredients: (Dershowitz'81); (Büchi'64); (McNaughton'94, Geser'01); automata completion (various authors)
- Independent implementations in Matchbox and MnM

Termination of (string) rewriting

Modular termination proofs by removing rules

- $\mathrm{SN}(R)$: R is *terminating* (or: *strongly normalizing*) if every R-derivation contains only finitely many R-steps.
- $\mathrm{SN}(R/S)$: R is *terminating relative to* S if every $(R \cup S)$-derivation contains only finitely many R-steps.
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How to prove SN(R), or prove SN(R/S)?

- Ad hoc approach: $0 \in$ finitely many.
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  Show that R-steps do not occur in any R-derivation, or
  show that R-steps do not occur in any $(R \cup S)$-derivation.
- Nonsensical, this is never the case . . .
  . . . but could work for a restricted set of derivations.

Restricting the set of derivations

Definition: Right-hand sides of forward closures

- $\mathsf{RFC}(R) = (\rightarrow_R \cup \rightharpoondown_{\mathsf{right}(R)})^*(\mathsf{rhs}(R))$,
  where $\rightharpoondown$ is suffix rewriting, and
  $\mathsf{right}(R) = \{\ell_1 \to r \mid (\ell_1\ell_2 \to r) \in R, \ell_1 \neq \epsilon \neq \ell_2\}$.

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R is terminating iff R is terminating on $\mathsf{RFC}(R)$.

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Example: $R = \{ab \to ba\}$

Here, $\mathsf{right}(R) = \{a \to ba\}$, so $\mathsf{RFC}(R) = (\rightarrow_R \cup \rightharpoondown_{\mathsf{right}(R)})^*(ba) = b^+a$.
$\mathsf{RFC}(R)$ contains no R-redex, so R is terminating.

Right barren string rewriting

Generalizing McNaughton'94, Geser'01
from 1-rule to arbitrary finite systems:

Definition: R is *right barren*

if no $\ell \in \mathrm{lhs}(R)$ is factor of a string in $\mathrm{RFC}(R)$.

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Proof of decidability

If R is right barren, $\mathrm{RFC}(R) = \rightharpoondown_{\mathrm{right}(R)}{}^*(\mathrm{rhs}(R))$. This set is regular, since regularity is preserved under suffix rewriting (Büchi'64).

Right barren string rewriting (cont'd)

Example: $R = \{babbaba \to abaabbabba\}$

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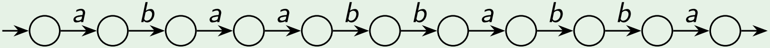
Automaton accepting rhs(R):



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Closure under $\rightharpoondown_{\mathrm{right}(R)}$ by adding epsilon transitions:

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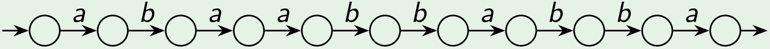

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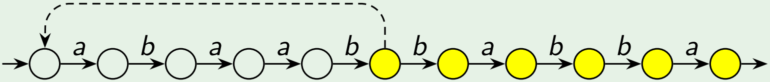

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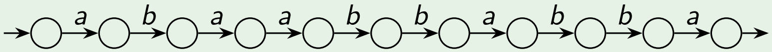

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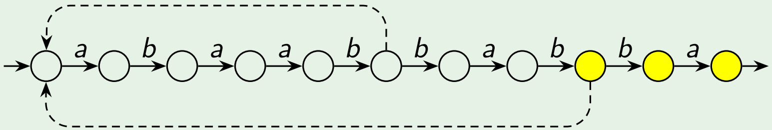

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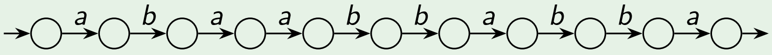

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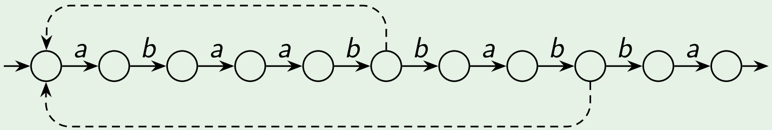
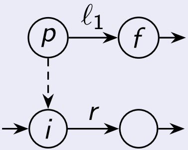


The left-hand side of R is not a factor of any accepted string,
so R is right barren, thus terminating.

Right barren string rewriting (cont'd)

Closure algorithm: suffix matches

For state p, final state f, $(\ell_1 \to r) \in \text{right}(R)$:
If there is a path $p \xrightarrow{\ell_1} f$, add $p \xrightarrow{\epsilon} i$,
where i is the initial state of the path for r.



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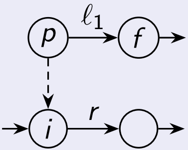

- Termination of this algorithm: No new nodes, so there are only finitely many possible epsilon transitions.
- Decide whether $\ell \in lhs(R)$ is a factor of some accepted string: check for path $p \xrightarrow{\ell} q$ (states are accessible and co-accessible).

Removing relatively right barren rules

Definition: $S \subseteq R$ is *relatively right barren* w. r. t. $R \setminus S$

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Then $\mathsf{SN}(R \setminus S)$ implies $\mathsf{SN}(R)$.

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Closure algorithm: suffix and redex matches

Closure steps for suffix matches as before.

Closure steps for redex matches:

For states p, q, and $(\ell \to r) \in R$:

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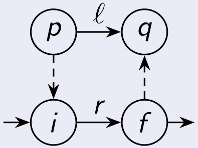

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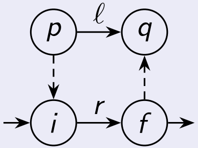

The resulting automaton over-approximates $\mathrm{RFC}(R)$.

Removing relatively right barren rules (cont'd)

Example: $R = \{ab \to ba, ba \to acb\}$ (`Zantema_04/z006`)

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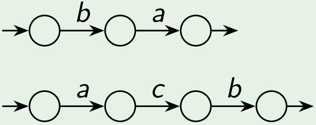

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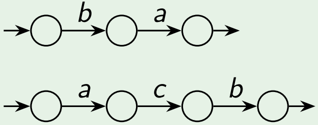

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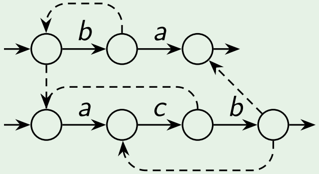

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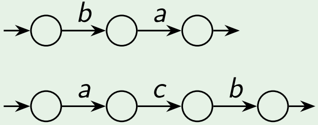

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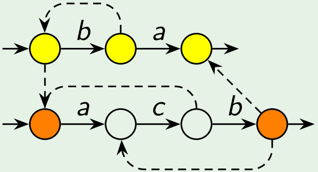

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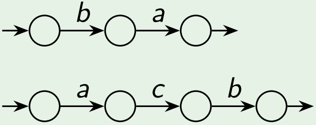

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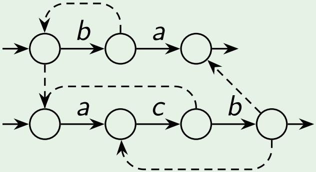

There is no path labelled by the left-hand side of $S = \{ab \to ba\}$:
S is relatively right barren w. r. t. $R \setminus S$. As $R \setminus S = \{ba \to acb\}$ is terminating (it is right barren), R is terminating.

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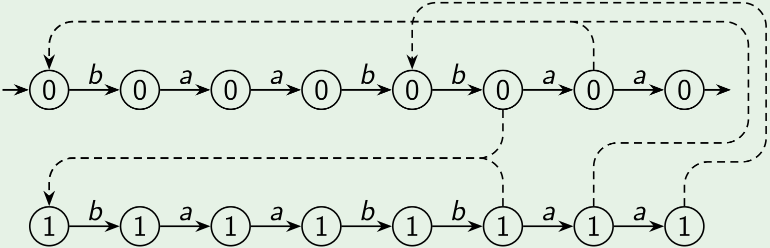
- In case of success: complete automaton is a certificate for match-bound B on RFC(R).

Approximating match-bounds (cont'd)

Example: $R = \{abaab \to baabbaa\}$ (`Zantema_04/z034` reversed)



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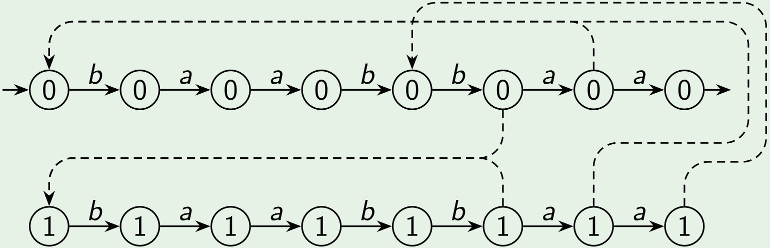

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Removing relatively match-bounded rules (sketch)

- Now layer B represents all heights $\geq B$; we never reject.
- After completion, remove those rules where all redex heights are $< B$: they are match-bounded relative to the remaining rules by B on RFC, so they terminating relative to the remaining rules.

Summary and discussion

- This method solves `SRS_STANDARD/ICFP_2010`.
  Weaker on non-`ICFP`: Solves 164 of 1056.
- Cannot solve `Zantema_04/z001`.
- But, by iteration, solves problems that are not (RFC-)match-bounded.
- Two independent implementations: Confidence, no certification.
- Combined with *drop common prefix/suffix*, nearly solves `Wenzel_16`: MnM solves 222 of 226.
- Implementation: keep the set of epsilon transitions transitively closed.
- Strategy: fix $B = 2$ or choose $B = 0, 1, \ldots$?

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- Challenge: merge this method with the exact RFC-method (Endrullis/H/W'06).
- Challenge: termCOMP needs more SRS benchmarks
  — that are independent of any specific method.
  Continue systematic or random enumeration.