Proving Non-Joinability using Weakly Monotone Algebras

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Motivation

- Def: *peak*: $s\ {}^*\!\leftarrow \cdot \rightarrow^* t$, *joinable*: $s \rightarrow^* \cdot {}^*\!\leftarrow t$
  confluent: each peak is joinable
- non-joinable: $\rightarrow^*(s) \cap \rightarrow^*(t) = \varnothing$.
  If $\rightarrow$ is non-terminating, then $\rightarrow^*(s), \rightarrow^*(t)$ can be infinite.
- . . . and need to be described in some finite way,
  e.g., as finite automata $A \supseteq \rightarrow^*(s)$, $B \supseteq \rightarrow^*(t)$.
  then check emptiness of $A \cap B$ (Zankl et al., 2011)
- this paper:
  - use weighted automata A, B,
    representing weakly monotone algebras,
  - such that Kronecker product algebra (represents
    $x \mapsto A(x) \cdot B(x)$) has bounded weights
  - such that bound is less than $A(s) \cdot B(t)$.

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Example (Ex. 1)

- non-joinability of ag, bh with respect to $\mathcal{R} = \{g \to ag, g \to i, h \to bh, h \to i, i \to abi, ab \to ba, ba \to ab\}$
- algebras $A : s \mapsto \#_a(s) - \#_b(s)$, $B : s \mapsto \#_b(s) - \#_a(s)$,
  - for $s \in \to_{\mathcal{R}}^*(ag) : 1 \leq A(s)$ — note: $(h \to bh)$ not usable
  - for $s \in \to_{\mathcal{R}}^*(bh) : 1 \leq B(s)$ — note: $(g \to ag)$ not usable
  - for all $s : A(ag) + B(bh) = 2 \not\leq 0 = A(s) + B(s)$
- cannot separate $\to_{\mathcal{R}}^*(ag)$ from $\to_{\mathcal{R}}^*(bh)$ with regular languages since:

$$\to_{\mathcal{R}}^*(ag) \supseteq \{a^n b^m i \mid n > m\}, \quad \to_{\mathcal{R}}^*(bh) \supseteq \{a^n b^m i \mid n < m\}$$

- represent A, B as arctically $(\{-\infty\} \cup \mathbb{Z}, \max, +)$ weighted automata, with one state each.
  Encode non-usability by $A(h) = -\infty$, $B(g) = -\infty$.

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Abstract Non-Joinability Criterion (Thm. 3)

- Let $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ be weakly monotone Σ-algebras such that $\mathcal{R}$ is weakly oriented by both $\mathcal{A}$ and $\mathcal{B}$, $s, t \in \mathcal{T}(\Sigma)$ be ground terms and $\delta : \mathcal{A} \times \mathcal{B} \to \mathcal{C}$ be a pre-homomorphism between weakly monotone Σ-algebras.
  Then s and t are non-joinable provided that for some $c \in \mathcal{C}$,
  1. $\delta([s]^{\mathcal{A}}, [t]^{\mathcal{B}}) \not\leq c$, and
  2. $f^{\mathcal{C}}(c, \ldots, c) \leq c$ for all $f \in \Sigma$.
- application (Ex. 7, compatible tree automata method)
  - A, B: finite automata; weakly oriented: $\mathcal{R}$-closed
  - C: their Cartesian product automaton (for intersection)
  - c: reachable states in C
- next: extend to weighted automata, restrict to strings

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Algebras from Finite Weighted Algebra

- $(S, \leq)$ a totally ordered semi-ring, e.g.,
  natural numbers $(\mathbb{N}, +, \cdot, 0, 1)$,
  arctic integers $(\mathbb{A}, \max, +, -\infty, 0)$, Booleans $(\mathbb{B}, \vee, \wedge, \mathbf{F}, \mathbf{T})$.
- S-weighted tree automaton A over alphabet Σ:
  - set of states Q,
  - family of transition mappings $\mu_k : \Sigma_k \to (Q^k \times Q \to S)$,
  - root weight vector $\nu : Q \to S$.

  The algebra μ_A of this automaton has domain $(Q \to S, \leq)$.
  (Q-indexed vectors of S values, ordered point-wise)
- Kronecker product automaton $A \odot B$ with states $Q_A \times Q_B$,
  $\mu_{A \odot B}(f)((v_A, v_B), (p_A, p_B)) = \mu_A(f)(v_A, p_A) \odot \mu_B(f)(v_B, p_B)$
- current implementation is for strings only,
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Implementation: Noko Leipzig

- for proving Nonkonfluenz (and it rhymes with a TV series)
- Noko Leipzig is part of Matchbox `https://gitlab.imn.htwk-leipzig.de/waldmann/pure-matchbox`
- core functionality: prove non-joinability
  - input: SRS $\mathcal{R}$ over Σ; $s, t \in \Sigma^*$; $d, b \in \mathbb{N}$.
  - output (if successful): arctically weighted automata A, B with d states, weights represented by b bits, and arctic vector $c \in (Q_A \times Q_B \to \mathbb{A})$, that fulfil the conditions of Theorem 3
- transform to a Boolean satisfiability problem with the Ersatz library (Kmett 201?), solve with Minisat (Sörensen 200?)
- performance in CoCo 2019 (for SRS): 6 unique NO answers, two (Cops 1034, 1131) using automata.

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- performance in CoCo 2019 (for SRS): 6 unique NO answers, two (Cops 1034, 1131) using automata.

An Example (21538)

- rules $R = R_1 \cup R_2$ where
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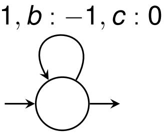
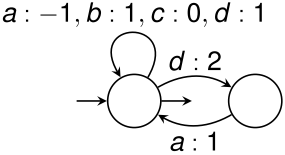
- $A =$ 

  , $B =$ 

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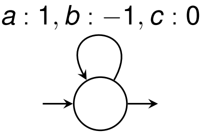
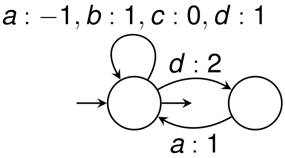
- $A =$ 

, $B =$ 

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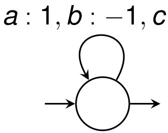
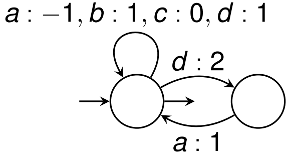
- $A =$ 

, $B =$ 

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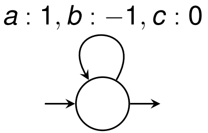
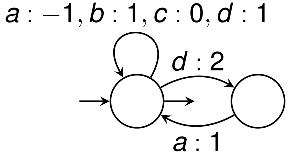
- $A =$ 

  , $B =$ 

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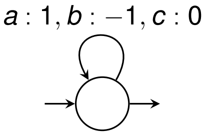
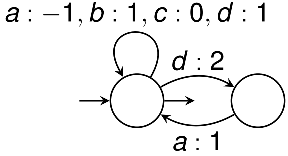
- $A =$ 

  , $B =$ 

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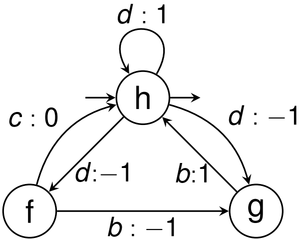
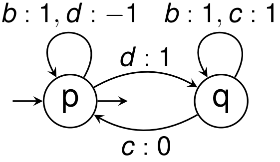
- $A =$ 

, $B =$ 

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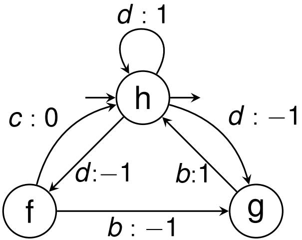
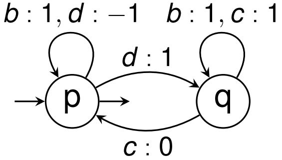
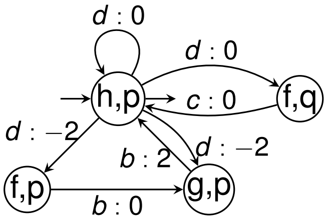
- $A =$ 

, $B =$ 

- $A \odot B =$ 

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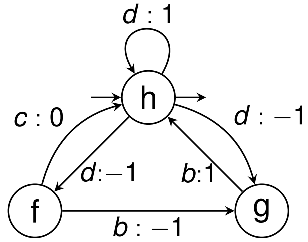
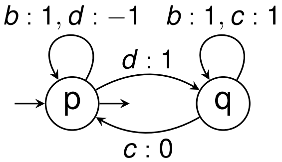
- $A =$ 

, $B =$ 

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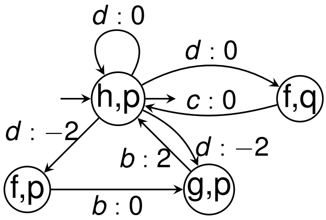
$A \odot B =$ 

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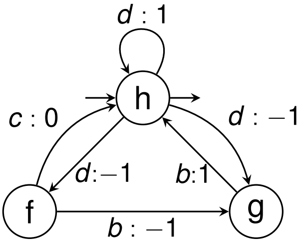
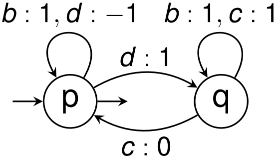
- $A =$ 

$, B =$ 

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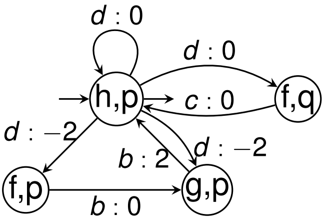
$A \odot B =$ 

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Related Work, Discussion

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  Both show that $\delta([s]^{\mathcal{A}}, [t]^{\mathcal{B}}) \not\leq \delta([u]^{\mathcal{A}}, [u]^{\mathcal{B}})$ for all u.
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