Strukturelle Induktion über Peano-Zahlen

```
data Nat = Z | S Nat

double :: Nat → Nat
double x = case x of
    Z   → Z
    S x' → S (S (double x'))

plus :: Nat → Nat → Nat
plus x y = case x of
    Z   → y
    (S x') → S (plus x' y)
```

Zu zeigen: $\forall x$: $\boxed{\text{double } x = \text{plus } x\ x}$

Induktion nach x

IA: $x = Z$, z.z. $\text{double } Z = \text{plus } Z\ Z$

$\boxed{\text{double } Z} \overset{\text{Def double}}{=} Z \overset{\text{Def plus}}{=} \boxed{\text{plus } Z\ Z}$

IS: $x = S\,x'$

IH: $\text{double } x' = \text{plus } x'\ x'$

z.z.: $\text{double } (S\,x') = \text{plus } (S\,x')\ (S\,x')$

Beweis: $\boxed{\text{double } (S\,x')} \overset{\text{Def double}}{=} S\,(S\,(\text{double } x')) \overset{\text{IH}}{=} S\,(S\,(\text{plus } x'\ x')) \overset{\text{Def plus}}{=} S\,(\text{plus } (S\,x')\ x') \overset{?}{=} S\,(\text{plus } x'\ (S\,x'))$

$= \boxed{\text{plus } (S\,x')\ (S\,x')}$