Strukturelle Induktion über Listen

$$\begin{tikzcd} xs \arrow[r, "\mathrm{sum}"] \arrow[d, "\mathrm{doubles}"'] & x \arrow[d, "\mathrm{double}"] \\ xs' \arrow[r, "\mathrm{sum}"] & x' \end{tikzcd}$$

```
data List a = Nil | Cons a (List a)

doubles :: ListNat → List~~Nat~~
doubles xs = case xs of
    Nil → Nil
    Cons x ~~xs~~ xss → Cons (double x) (doubles xss)

sum :: List Nat → Nat
sum xs = case xs of
    Nil → z
    Cons x xss → plus x (sum xss)
```

zu zeigen: ~~∀xs~~ $\boxed{\mathrm{sum}\,(\mathrm{doubles}\ xs) = \mathrm{double}\,(\mathrm{sum}\ xs)}$

Induktion über xs

IA: xs = Nil

z.z.: $\mathrm{sum}(\mathrm{doubles}(\mathrm{Nil})) = \mathrm{double}(\mathrm{sum}(\mathrm{Nil}))$

Beweis: $s(ds(\mathrm{Nil})) \overset{\mathrm{Def}\ ds}{=} s(\mathrm{Nil}) \overset{\mathrm{Def}\ s}{=} z \overset{\mathrm{Def}\ d}{=} d(z) \overset{\mathrm{Def}\ s}{=} d(s(\mathrm{Nil}))$

IS: xs = Cons x xss

IH: $\boxed{s(ds)}(xss) = \boxed{d(s)}(xss)$

z.z. $\boxed{s(ds)}(\mathrm{Cons}\ x\ xss) = \boxed{d(s)}(\mathrm{Cons}\ x\ xss)$

Beweis: $s(ds(\mathrm{Cons}\ x\ xss)) \overset{ds}{=} s(\mathrm{Cons}(d(x), ds(xss))) \overset{s}{=} +(d(x), \boxed{s(ds(xss))}) \overset{IH}{=} +(d(x), d(s(xss))) \overset{\text{üA}}{\overset{dp}{=}} d(+(x, s(xss))) \overset{s}{=} d(s(\mathrm{Cons}\ x\ xss))$